What is the role of sacrifices in the Old Testament? How did Old Testament sacrifice relate to the cross?

While salvation has always been by God's grace through the work of Christ on the cross, on the human recipient the requirement and result of that grace has been faith in God's revelation promise of deliverance.

Adam and Eve were saved through faith in the promise of deliverance revealed to them [Genesis 3.15], which was that God in time would bring about a righteous descendent of the woman who would trust and obey God completely, thereby representing God well and restoring rule to man instead of Satan. The first evidence of their faith is that Adam named his wife "Life" [Genesis 3.20] just after the pronouncement of death [Genesis 3.19]. They accepted from God the provision of skins from the first sacrificed animals [Genesis 3.21]. This symbolized the sacrificial system that would cover sin until the promised New Adam came as the ultimate sacrifice, and represented that God would continue to provide for the needs of his children. Further evidence of their faith in the revealed promise was that when her first child came, Eve said literally, "I have gotten a man-child, Yahweh" [4.1]: she thought this child was the promised seed of the woman who would set things right.

Abel responded in faith to this promise by acting righteously and sacrificing animals as God had directed [Genesis 4.4]. At the birth of his son Noah, Lamech stated he thought Noah would be the one to bring rest from the toil resulting from God's curse of the ground [Genesis 5.29], i.e. that Noah would be the Genesis 3.15 deliverer. Noah did deliver in a limited sense: God spared Noah, his family, and two of every species, apparently because of Noah's distinctive willingness to obediently act on revelation [Genesis 6.8].

With Abraham, there came further revelation, as God chose Abraham to be the one to establish a line through which would come the promised Savior [Genesis 12.1-3]. This was a modification of the Genesis 1.26-28 promise: that earlier promise was that man would represent God in ruling the earth, with the later addition of a family unit to support the man in this endeavor [2.22-24]; this new promise of Genesis 12.1-3 was that from one man would come a family that would develop into a nation, and that nation would represent God in ruling the rest of mankind. God used many experiences to bring Abraham to a full measure of faith, such that Abraham came to fully trust and obey God [22.1-18].

The ultimate test of Abraham's faith was when God commanded him to sacrifice his beloved and promised son, Isaac, as part of the Genesis 3.15 promise, and Abraham obeyed and trusted that God would still deliver on his promise of Genesis 12.1-3 [22:1-18]. Isaac was to be the righteous sacrifice [Genesis 22], the deliverer for the nation from its sin and rule by Satan, righteously being obedient to God even unto death. However, this was a faith building exercise, for Isaac was not the true Genesis 3.15 promised "seed" of the woman, so God stayed Abraham's hand to prevent the sacrifice, and provided a ram for the sacrifice instead [22.11-13]. The true ultimate "seed" who would deliver on the Genesis 3.15 promise would come later in Christ.

Even though it was from Abraham that God would raise a family into a nation to represent him on the earth, because Abraham was sinful there was a need for a mediator between him and God, as represented in the Genesis 3.15 promise of a righteous one who would deliver the people from sin and Satanic rule. Melchizedek [14.18-20] represented that mediation. Abraham recognized this in faith, and accepted Melchizedek's blessing while offering tithes.

So there were two strains of sacrifice-theology evident prior to the Law of Moses: there was the promise of ultimate deliverance through a Savior descended from Eve and a system of animal sacrifices that had become a part of this promise and part of obedience. Acting on these constituted faith in God's promise of ultimate deliverance of man.

With the Law of Moses, the sacrificial system become codified. The Law also served to further establish God's character and thus his expectations for man, to reveal the reality of sin and the need for deliverance. As part of this legal system, the sacrifices demonstrated the magnitude of consequences of sin: death. The death of the innocent animal was a temporary covering for the sin of man, symbolizing the coming sacrifice of the ultimate deliverer and delaying God's judgment on man.

The sacrifices were atoning, which is to say they brought man back into unity with God by appeasing God's anger and delaying his judgment, a foreshadowing of the atonement Christ would accomplish on the cross, which would provide for imputed righteousness through justification [God accounting to us the righteousness of Christ], by which the relationship between a righteous God and sinful man was made sustainable. The sacrifices were an act of faith in the promises of ultimate deliverance through the Savior and of temporary deliverance from the penalty of death.